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Foundation Mathematics

Topic 9 – Lecture 2: Introducing Probability

Using Tree Diagrams to Determine Probability

Combinations and Permutations

V1.0

Scope and Coverage

This topic will cover:

- The use of diagrams to represent probabilities
- The principles underlying permutations and combinations and their relationship to probability

Learning Outcomes

By the end of this topic students will be able to:

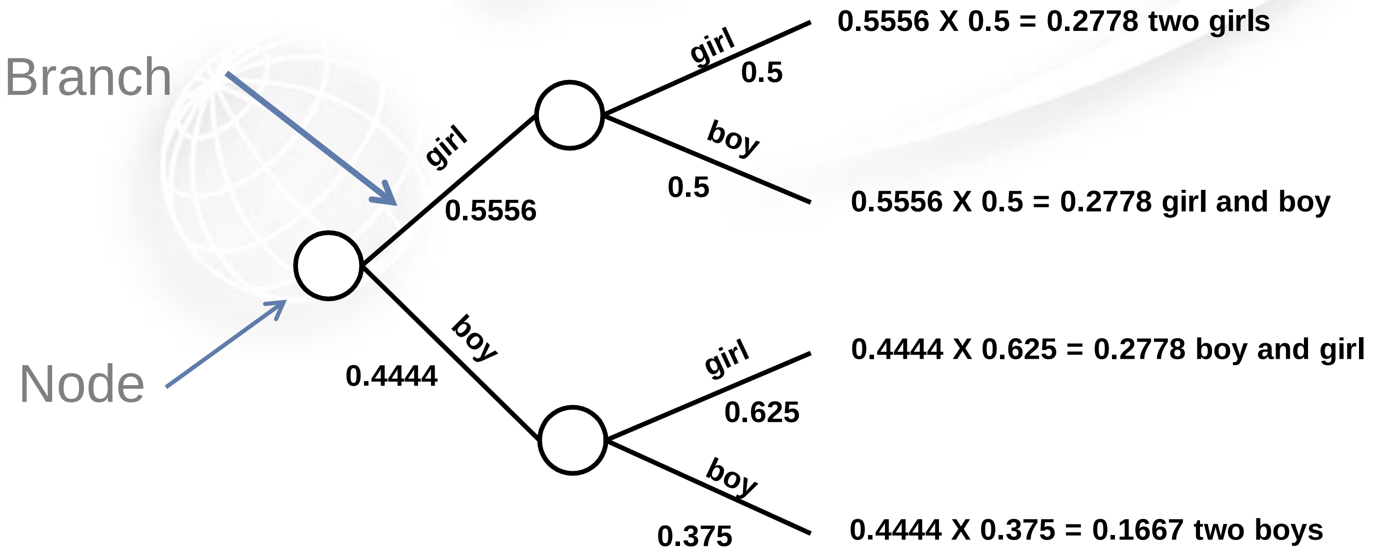
- Calculate probabilities using tree diagrams
- Calculate permutations and combinations

Tree Diagrams - 1

- It is sometimes more efficient to show probabilities between one event and another by using tree diagrams.
- Consider:
  - If two students are to be chosen randomly from a group of 5 girls and 4 boys then:
  - The probability that the first person chosen is a girl = 0.5556
  - The probability that it is a boy = 0.4444
- We know that we can calculate the probabilities of choosing a boy from the class if we have already chosen a girl and that such an outcome is a dependent relationship.

Tree Diagrams - 2

- For the previous example we can consider the probability of all selection events and place them in a tree diagram.



- The diagram show us the event that is happening and the associated probability

Tree Diagrams – Example - 1

- The demand for gas is dependent on the weather and much research has been undertaken to accurately forecast the demand.
- This is important since it is quite difficult and expensive to increase the supply at short notice. If on any particular day the air temperature is below normal, the probability of high demand occurring is 0.6
- However, at normal temperature the probability of a high demand occurring is only 0.2 and if the temperature is above normal the probability of a high demand drops to 0.05
- What is the probability of high demand occurring if over a period of time the temperature is below normal 20% of occasions and above normal 30% of occasions?

Tree Diagrams – Example - 2

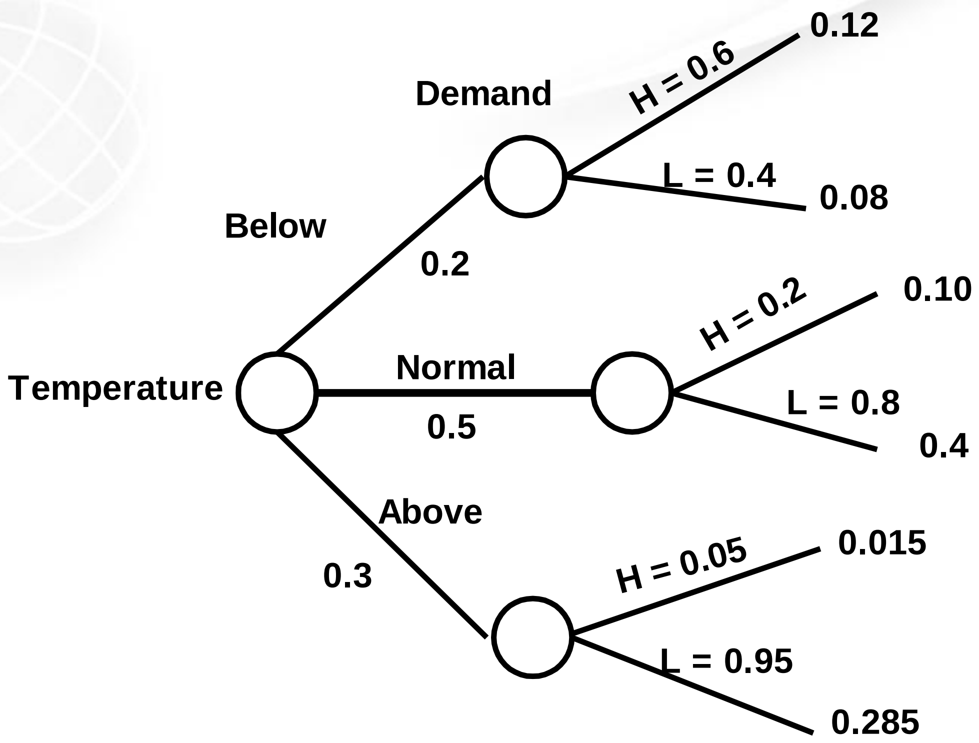
- Since the demand depends on temperature the first node refers to temperature and as such there are three branches:
- What is the probability of a high demand occurring if over a period of time the temperature is below normal 20% of occasions and above normal 30% of occasions.
- Since demand depends on temperature the first node refers to temperature and there are three branches:
  - Below normal
  - Normal and
  - Above normal

Tree Diagrams – Example - 3

The probability of a normal temperature is:

1 - (below temperature + above temperature) = 1 - (0.2 + 0.3) = 0.5

We can now use this information to construct our probability tree



Tree Diagrams – Example - 4

- The compound probability for each route is written at the end of the route; therefore the probability of there being a high demand given that the temperature is below normal is:
  - 0.2 X 0.6 = 0.12
- Since there are three possible routes where the demand could be high, the law of addition is used and the probability is:
  - 0.12 + 0.10 + 0.015 = 0.235

Combinations 1

- A combination is a set of items, selected from a large collection of items, regardless of the order in which they are selected.
- We refer to the possible combinations of r items from n unlike items.
- Example
  - Suppose that five people apply for two vacancies as accountants in Combo Ltd, and these people are referred to as: A B C D E
  - What possible combinations of applicants could be employed?

Combinations 2

- The different possible combinations of people to fill the two posts from the five applicants would be
  - AB, AC AD AE, BC, BD, BE, CD, CE and DE.
- This way of identifying combinations is possible only due to the fact that there are relatively small numbers of items and therefore possible combinations.
- Although for such a small sample identifying the combinations is not that difficult, the complexity of such a task becomes much greater when we start to deal with much larger data sets.

Combination Formula

- To overcome the difficulties that can be associated with large amounts of data and therefore large numbers of possible combinations we can apply the following formula

$$nCr = \frac{n!}{(n-r)!r!}$$

- The notation nCr means the number of different combinations of r items which are possible from a set of n unlikely items.
- The symbol ! is in mathematics referred to as **factorial** and is simply the multiplication of all numbers from the number which has the ! symbol next to it to 1.
- Example 4! = 4 X 3 X 2 X 1

Combination Formula Continued

- In the previous example for five candidates applying for two jobs the number of possible combinations of two applicants would be

$$ {}_5C_2 = \frac{5!}{(5-2)!(2!)} = \frac{5\times4\times3\times2\times1}{(3\times2\times1)(2\times1)} = 10 $$

- Therefore there are 10 possible combinations that can occur as a consequence of our five candidates ABCDE

Permutations

- The approach to calculating permutations is very similar to that of combinations

- A permutation is a set of items, selected from a larger group of items, in which the order of selection or arrangement is significant. We refer to the number of possible permutations of r items from n unlike items.

Permutations – Example - 1

- If we consider from our previous example that the two positions available are those of Manager and Secretary we get the following permutations

Manager	Secretary	Manager	Secretary	Manager	Secretary	Manager	Secretary
A	B	B	C	C	D	D	E
A	C	B	D	C	E	E	A
A	D	B	E	D	A	E	B
A	E	C	A	D	B	E	C
B	A	C	B	D	C	E	D

Permutations – Example - 2

- Whereas there are 10 combinations of two from five, there are twenty permutations. As can be seen from the table, calculating all possible permutations becomes a complex, time consuming task.
- However, we can apply a relatively simple formula for calculating the possible number of permutations.
- The formula for calculating permutations is as follows:

$$\mathrm{nPr} = \frac{n!}{(n-r)!}$$

Permutations – Example - 3

- Therefore by substituting values into our equation we get

$$ {}_5P_2 = \frac{5!}{(5-2)!} = \frac{5\times 4\times 3\times 2\times 1}{3\times 2\times 1} = 20 $$

- Thus the application of the formula can save lots of time and effort when determining permutations
- Remember 0!=1

Topic 9 – Introducing Probability 2

Any Questions?