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Foundation Mathematics

Topic 9: Introducing Probability

The Addition and Multiplication Rule

Probability of Compound Events

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Scope and Coverage

This topic will cover:

- An introduction to the underlying concepts of probability
- Calculation of probabilities of events using the addition and multiplication rules
- Calculation of probabilities of compound events

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Learning Outcomes

By the end of this topic students will be able to:

- Calculate probabilities using the addition and multiplication rule
- Calculate probabilities of compound events

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Introducing Probability

- The concept of **probability** occupies an important place in the **decision-making** process, whether the problem is one faced in business, in government or just in everyday personal life.
- In very few decision-making situations do we have **perfect information** i.e. all the needed facts - available.
- Most decisions are made in the face of **uncertainty**. Probability enters into the process of decision making by playing the role of a **substitute for certainty** - a substitute for complete knowledge.

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Probability – Concepts - 1

- Example
  - A local weather forecast gives us “a fifty percent chance of rain this afternoon”. Therefore we have a numerical probability of rain 50%
- How should we plan our response to this?
- Are we prepared to accept a 50% chance of rain as acceptable and proceed with our barbeque?
- If the other 50% chance is of snow then perhaps not
- If the other 50% is for sunny weather then may be we will take that chance?

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Probability – Concepts - 2

- In achieving a numerical value of probability we are able to make decisions based upon that probability value
- In terms of defining probability we can think of it as ‘a likelihood of an event that can be expressed in a numerical format’
- Example – if you toss a coin into the air and let it fall to the ground you have an equal chance of getting a head or a tail
- If we represent this in mathematical terms
  - The probability of a head is $\frac{1}{2}$ or 0.5
  - The probability of a tail is $\frac{1}{2}$ or 0.5
  - The probability of either a head or a tail is 1.0

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Probability Concepts - 3

- Important concepts:
- The probability of an event is presented as a fraction of 1.0
- Some events are mutually exclusive; we couldn't get both a head and tail on the toss of a single coin
- If an event is impossible it has the probability of 0
- If an event is certain to happen then it has a probability of 1.0

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Mutual Exclusivity - 1

- As already stated some events cannot take place at the same time and are therefore mutually exclusive. Another way of thinking of this is that if one event takes place it excludes the possibility of the other event happening
- Consider the roll of a dice – What is the probability of getting a 3 or a 4?
  - Probability of a 3 = $\frac{1}{6}$
  - Probability of a 4 = $\frac{1}{6}$
  - The probability of rolling a 3 or a 4 = $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

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Mutual Exclusivity - 2

- Representing this in a general equation for two mutually exclusive events A and B we can present the probability of A or B occurring as
  P(A) or P(B) = P(A) + P(B)
- Note: this is for a single event. In our example we only rolled the dice once and we therefore considered the probabilities of getting either a 3 or a 4 for this event only.

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Non Mutually Exclusive Events - 1

- Two (or more) outcomes are possible at the same time
- Example – if we have a standard pack of 52 playing cards what is the possibility of drawing either a Picture Card P(A) or a Heart P(B)
- A pack of 52 cards will have 13 Hearts and 12 Picture Cards
- Our events are ***not mutually exclusive*** as it is possible to draw a card which is ***both*** a Heart and a Picture Card

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Non Mutually Exclusive Events - 2

- Applying our earlier probability rule we get
  P(A or B) = $\frac{13}{52}+\frac{12}{52}$

- However of the 13 Heart cards in the pack the King, Queen and Jack are picture cards too so to prevent double counting we must subtract these from our P(A or B)

- Therefore P(A or B) = $\frac{13}{52}+\frac{12}{52}-\frac{3}{52}=\frac{22}{52}$

- As an equation P(A or B) = P(A) + P(B) – P(AB)

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Venn Diagrams

- Although we have looked at the probabilities of mutually exclusive and non mutually exclusive events in mathematical forms it is possible to represent these as ***Venn Diagrams***
- A Venn diagram is usually drawn as a rectangle the inside area of the rectangle represents to all possible events under consideration.
- Within the diagram the probabilities of events may be shown as circles which sometimes remain separate and at other times overlap.

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Using Venn Diagrams - 1

- Consider the two events A and B:
  - A is the probability of drawing a Jack from a standard pack of cards.
  - B is the probability of drawing an Ace from a standard pack of cards.

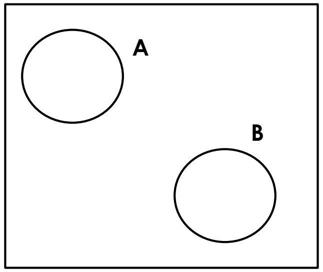
- The probability of drawing either a jack or an Ace is:

P(Jack or Ace) = P(Jack) + P(Ace)

= 0.0769 + 0.0769

= 0.1538

- This can be represented in Venn diagram form as:



Where A = Jack, B = Ace

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Using Venn Diagrams - 2

- However, if we consider the possibility of drawing
  - A Jack
  - B Diamond
- Then these two events actually overlap and if we simply add the probabilities together we actually double count the Jack of diamonds in our calculation so to remove this double counting activity the following expression can be written:
  - P(Jack or Diamond) = P(Jack)+ P(Diamond) – P(Jack of Diamonds)
  - P(Jack or Diamond) = (0.0769) +(0.25) – P(0.0192) = 0.3077

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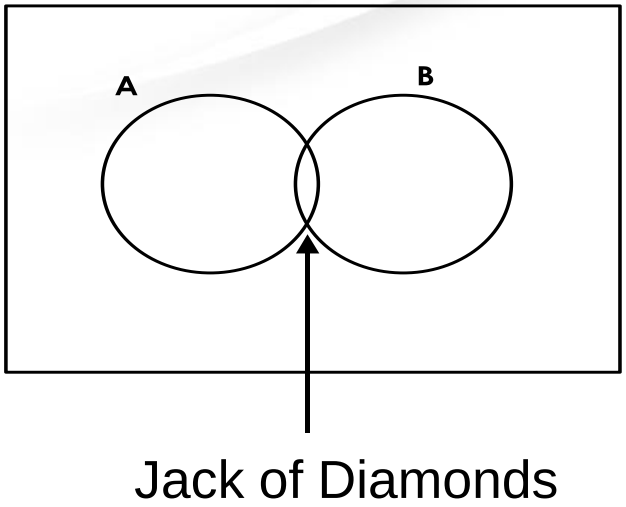
Using Venn Diagrams - 3

- The probability of drawing a Jack of Diamonds is the area of the overlap between the two circles. The probability of drawing a Jack of Diamonds is much less that the probability of drawing a Jack or a Diamond



- The equation P(A or B) = P(A) + P(B) – P(AB) is known as the
- General Addition Rule of Probability.

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Independent Events

- If the occurrence of one event has no effect on another event then these are ***independent events***. For example:
  - For a normal pack of playing cards what is the probability of drawing a Jack, putting the card back and then drawing a King?

$$P(J)=\frac{4}{52}=\frac{1}{13};P(K)=\frac{4}{52}=\frac{1}{13}$$

- Since we have independent events we must multiply the probabilities of each individual event

$$P(J) and P(K)=\frac{1}{13}\times\frac{1}{13}=\frac{1}{169}$$

- Therefore for two independent events A and B the probability of A and B occurring can be written as P(A and B) = P(A) x P(B)

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Conditional Probability

- In these circumstances we look at probabilities of events that depend in part on whether or not other events have taken place.
- If the probability of an event occurring is dependent on whether a previous event (Event A) has occurred then it is possible to say that event B is conditional on event A.
- Representing this statement mathematically is done thus: $(B \mid A)$

  which simply means the probability of B given that A has already occurred.
- Where events are ***independent*** $(B \mid A) = B$
- ***Sampling without replacement*** is a good example of ***conditional probability***

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Conditional Probability - Example

- If two students are to be chosen randomly from a group of 5 girls and 4 boys then the probability that the first person chosen is a girl is $\frac{5}{9}$ or 0. 5556 and the probability it is a boy is $\frac{4}{9}$ or 0.4444
- If after choosing a person they are removed from the group this will affect the probability of whether or not a boy or girl will be chosen next
- If a girl was chosen first then the probability of choosing a girl at the next event would be $\frac{4}{8}$. As we can see the numerator has decreased by 1 (number of girls) and the denominator (class size) by 1 also
- The probability of choosing a boy is $\frac{4}{8}$

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Conditional Probability

- If we want to know the probability of the first student being a girl and the second student being a girl then we need to use the multiplication rule.

  So event A=girl, event B=girl

  P(A and B) = P(A) X P(BIA)

  P(girl and a girl) = 0.5556 X 0.5

  = 0.2778

- In both these events the students have been selected from the class and not replaced into the population.
- If we replace the student back into the population then neither the numerator nor denominator change in value.

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Bayes Theorem

- Bayes Theorem states:
  - ‘The probability of two dependent events occurring can be expressed as the probability of the first event occurring multiplied by the conditional probability of the second event occurring given that the first has already occurred.’
- As an equation P(B I A) = $\frac{P(B) \times P(A| B)}{P(A)}$
- Therefore the probability of B given A = $\frac{P(BandA)}{P(A)}$

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Any Questions?

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