Bringing British Education to You

www.nccedu.com

Foundation Mathematics

Topic 7 – Lecture 1: Beginning Statistics

Measures of Central Tendency

V1.0

Scope and Coverage

This topic will cover:

- Calculation of the arithmetic mean for a range of data samples
- Calculation of the arithmetic mean for a range of frequency distributions

Learning Outcomes

By the end of this topic students will be able to:

- Calculate the arithmetic mean of data samples
- Calculate the arithmetic mean for a range of frequency distributions
- Calculate the estimated mean from grouped data

Measures of Central Tendency

- Two initial measures that we can take:
  - A measure of centrality - an average
  - A measure of dispersion - a spread
- For this discussion we will focus on the measure of central tendency, or average.
- Finding an averages is a method of determining the ‘location’ of a central point of a distribution. As such it is a figure used to give some impression of the size of all the items in a population.

Averages

- Although averages can be used to give a rough estimation of figures within a population they will not give information about the dispersion of values in a distribution.
- Although the term ‘average’ is one which is commonly used in everyday business discussions, there are actually three main types of average:
  - The Mean
  - The Median
  - The Mode

The Arithmetic Mean

- This is probably the most commonly recognised average and is routinely used in the simplification and management of data.
- The more common name of the arithmetic mean is just simply “the mean” and therefore it will be referred to as this throughout our discussions.
- The calculation of the arithmetic mean is found by the following formula

$$\bar{x} = \frac{\sum x}{n}$$

- Where $\bar{x}$ is the notation for the mean, Σ indicates the sum of the variable x and n is the size of the data set.

The Mean

- To illustrate how the mean is calculated consider the data set below

1
2
3
4
5
6
7
8
9
45

$n = 9$

- By simply dividing our total for $x\,(\sum x)$ by the total number (n) we get $\frac{45}{9} = 5$ therefore our mean or $\bar{x} = 5$

The Mean – Benefits and Drawbacks

- Advantages
  - It is easy to calculate
  - It is widely understood
  - The value of every item is included in the calculation of the mean
  - It is supported by mathematical theory and is suited to further statistical analysis
- Disadvantages
  - Its value may not correspond to any actual value. For example the average family might have 2.3 children (impossible).
  - The arithmetic mean is distorted by extremely high or low values.

The Mean – Frequency Distributions - 1

- When large quantities of data are required or when the data is provided in the form of a frequency table the method of calculation is a little harder.
- The following data is the number of daily sales of boots in a women's shoe shop:

Number of Sales	Frequency
2	3
3	7
4	9
5	6
6	5
7	2
8	1

To calculate the mean we simply cannot add up our number of sale as these are linked to the frequency of sales.

The Mean – Frequency Distributions - 2

- To calculate the mean for our frequency distribution we present our data in a different format

Number of Sales (x)	Frequency (f)	fx
2	3	6
3	7	21
4	9	36
5	6	30
6	5	30
7	2	14
8	1	8
Total	33	145

- fx is simply the number of sales multiplied by the frequency

The Mean – Frequency Distributions – 3

- The calculation of the mean can then take place which is simply $\frac{145}{33} = 4.4$ sales.
- Therefore the mean sales per day is 4.4 (note this can be given as a decimal even though it is unlikely to sell only part of a boot)
- If we want to express this method for calculating the mean in algebraic notation we do so as

$$\bar{x} = \frac{\sum f x}{\sum f}$$

The Mean - Grouped Data - 1

- When we simplify data to make it more presentable for analysis we actually increase the level of complexity surrounding the calculation of useful statistics such as the mean
- For ease of management it is often found that data are grouped together, as below. As grouped data is a simplified representation, the exact mean cannot be found. Instead we calculate the **estimated mean**.

Age Range	Frequency
20 – 29	6
30 – 39	8
40 – 49	9
50 – 59	5
60 – 69	3

What we need to do now is manipulate such data to enable us to work out average age of the population.

The Mean – Grouped Data - 2

- To undertake such a calculation we need to present our information in a modified table
- Our age range (***this is referred to as our class interval***) is the same for all (9) therefore the mid point is 4.5 above the lower value of the class interval.

Age Range	Mid point of range (x)	Frequency (f)	fx
20 – 29	24.5	6	147.00
30 – 39	34.5	8	276.00
40 – 49	44.5	9	400.50
50 – 59	54.5	5	272.50
60 – 69	64.5	3	193.50
	total	31	1289.50

$$\frac{\Sigma fx}{\Sigma f} = \frac{1289.50}{31} = \mathbf{41.6}$$

Grouped Data

- Consider the following example – An automated potato packaging machine is measured to determine the average weight of potatoes packed during one day. Measurements were taken and the following data collected. What is the estimated mean value of potatoes packed by the machine?

Weight (kg)	Frequency of observation
$0 \le w < 10$	2
$10 \le w < 20$	3
$20 \le w < 30$	7
$30 \le w < 40$	15
$40 \le w < 50$	8
$50 \le w < 60$	4
$60 \le w < 70$	1

Grouped Data – Example Continued

- To calculate our mean value $\overline{X}$ we need to order our table to get the relevant information.

Weight (kgs)	Midpoint X	Frequency f	f X
0 ≤w< 10	5	2	10
10 ≤w< 20	15	3	45
20 ≤w< 30	25	7	175
30 ≤w< 40	35	15	525
40 ≤w< 50	45	8	360
50 ≤w< 60	55	4	220
60 ≤w< 70	65	1	65
		40	1 400

Grouped Data

- By presenting our information in such a way it is relatively easy to determine $\frac{\sum f x}{\sum f}$
- Which by substituting in our values gives

$$\frac{1400}{40} = 35kg$$

- If this is within an acceptable range for the machine then no modifications to its operation are needed. However, if not then we have evidence to suggest that it is not working as it should.

Topic 7 – Beginning Statistics

Any Questions?

Bringing British Education to You

www.nccedu.com