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Foundation Mathematics

Topic 5 – Lecture 2: Introduction to Integral Calculus

The Definite Integral

The Area Under a Curve

Scope and Coverage

This topic will cover:

- An evaluation of the definite integral
- Calculation of the area under a curve

Learning Outcomes

By the end of this topic students will be able to:

- Recognise the definite integral and apply it to a range of algebraic functions
- Calculate the area under a curve by using integration

The Definite Integral

- When we looked at performing integration, we followed a process that converted our function
- $\int x^n dx$ into the form $\dfrac{x^{n+1}}{n+1} + c$
- We also recognised that in performing this process we need to include the constant **c**
- This result is called an **indefinite integral** and the constant included is **arbitrary**; therefore it can take a number of different numerical values.

The Definite Integral – Applying Limits

- For many purposes we require definite integrals, which are written in the form $\int_b^a x^n dx$
- As you can see there are letters (a and b) associated with the integration sign.
- The values of a and b are called the ***limits***, a being the ***upper limit*** and b the ***lower limit***.

Calculating the Definite Integral – 1

- By introducing limits to the process of integration, we need to perform a different process that will help us calculate the ***definite integral***

- Example: Find the value of $\int_2^3 x^2 dx$

- We have limits associated with our function of integration, with the lower limit set at 2 and an upper limit of 3

- By integrating our expression $\int_2^3 x^2 dx$ we get = $\left[\frac{x^3}{3}\right]_2^3$

Calculating the Definite Integral – 2

- Inset the limit values into our expression:
- (value of $\frac{x^3}{3}$ when x is put equal to 3) – (value of $\frac{x^3}{3}$ when x is put equal to 2)
- This then gives us $\frac{3^3}{3} - \frac{2^3}{3} = \frac{27}{3} - \frac{8}{3} = \frac{19}{3} = 6\frac{1}{3}$

The Definite Integral - Complex Expressions - 1

- Example: Find the value of $\int_1^2 (3x^2 - 2x + 5)dx$
- We have an expression containing x raised to a power (x^2), an x term with coefficient 2 and a numerical value, 5.
- If we first integrate we get the following

$$\int_1^2 (3x^2 - 2x + 5) = \left[x^3 - x^2 + 5x\right]_1^2$$

The Definite Integral - Complex Expressions - 2

- As we have an integral with both upper and lower limits we need to substitute the limits into the integral.

$$\left[x^3 - x^2 + 5x\right]_1^2 = (2^3 - 2^2 + 5\times 2) - (1^3 - 1^2 + 5\times 1)$$

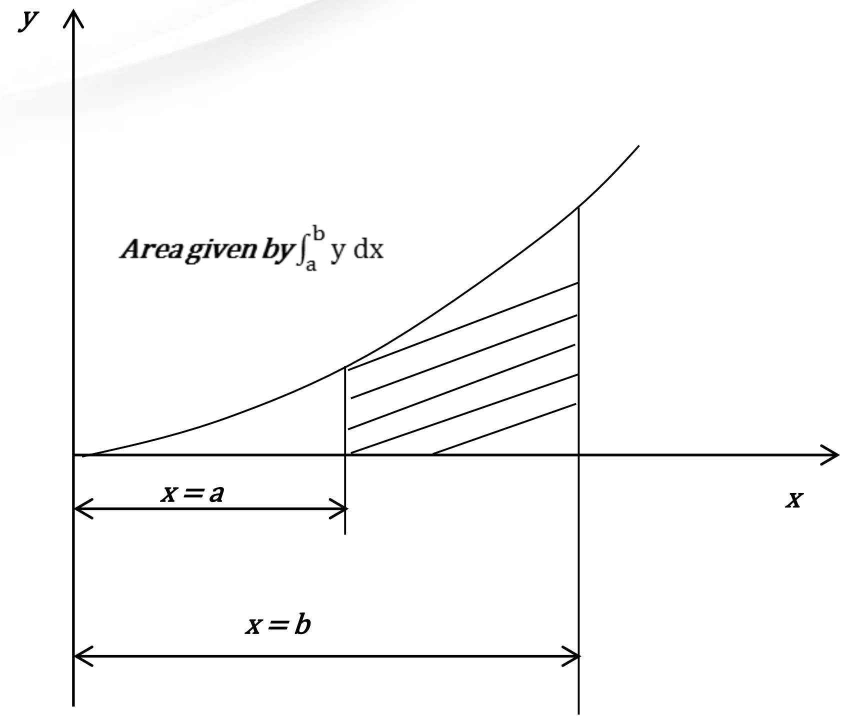
- Calculating the numerical values after we have substituted out limits for values of x we get:

14 – 5 = **9**

The Area Under a Curve - 1

One of the applications of integration using limits is to be able to calculate the area under a curve.

In the diagram, we can see that the area under the curve with shaded lines has limits and is contained within the part of the curve between x= b and x = a



The Area Under a Curve - 2

- In mathematical terms we can see that if we want to find the area under a curve, we can use the fact that the area is contained within the limits x = a and x = b, our curve and the x axis plus our knowledge of integration.
- In general, to do this we use integration with limits which looks like:

$$\int_b^a y \; dx$$

The Area Under a Curve - 3

Example: Find the area bounded by the curve $y = x^3 + 3$, the x axis and the lines x = 1 and x = 3

We organise our information ready for integration

A = $\int_1^3 (x^3 + 3)dx$ by integrating we get = $\left[\frac{x^4}{4} + 3x\right]_1^3$ now by

substituting in our limits, it becomes

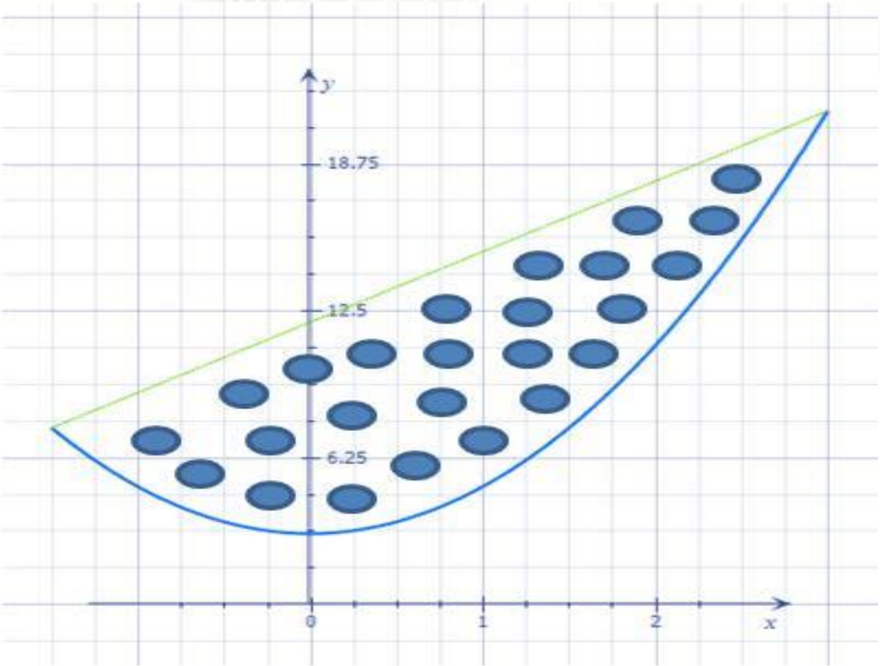
$$\left[\frac{3^4}{4} + 3 \times 3\right] - \left[\frac{1^4}{4} + 3 \times 1\right]$$

The Area Under a Curve - 3

Simply working these out gives $29\frac{1}{4} - 3\frac{1}{4} = 26$ square units

The Area Bounded by a Curve - 1

So far we have looked at curves that are bound by limits and also the x axis. However we can also use this approach to calculate other areas which have curves as boundaries



The straight line has the equation
$y = 12 + 3x$

The curve has the equation
$y = 2x^2 + 3$

What is required is to find the shaded area between the straight line and curve?

The Area Bounded by a Curve - 2

By applying integration, we can calculate this area which as we have both straight lines and curves would be very difficult to do by other mathematical methods or even measuring!

First find the points of the intersection of the line and curve. At this point the equations of the straight line and curve will be equal to each other, therefore:

$2x^2 + 3$ (equation of the curve) = 12 + $3x$ (equation of the straight line)

The Area Bounded by a Curve - 3

If $2x^2 + 3 = 12 + 3x$ then we can rearrange to get

$2x^2 - 3x - 9 = 0$ and this can be easily factorised to give

$$(2x+3)(x-3) = 0$$

Therefore by calculation our values of x are x = - 1.5 and 3.

These values are important as they are the limits that we will apply when calculating our area by integration.

The Area Bounded by a Curve - 4

As we are trying to calculate the area between the straight line and the curve we need to take both equations into account when integrating, therefore:

The shaded area = area under the line – area under the curve

Expressing this in mathematical terms gives us:

$$\int_{-1.5}^{3} (12+3x)dx - \int_{-1.5}^{3} (2x^2+3)dx$$

The Area Bounded by a Curve - 5

Taking the equation of the straight line first we get

$$\int_{-1.5}^{3}(12+3x)dx=\left[12x+\frac{3x^2}{2}\right]_{-1.5}^{3}$$

= (36 + 13.5) – (-18 + 3.375) which gives us a total of ***64.125 square units***

Taking the equation of the curve gives

$$\int_{-1.5}^{3}(2x^2+3)dx=\left[\frac{2x^3}{3}+3x\right]_{-1.5}^{3}$$

= (18 + 9) – (-2.25 – 4.5)

= 33.75 square units

The Area Bounded by a Curve - 6

- Therefore the ***shaded area*** is ***calculated*** as the area under the line – area under the curve

64.125 square units – 33.75 square units

= 30.375 square units

- This approach allows us to calculate complex areas through integration.

Topic 5 - Introduction to Integral Calculus 2

Any Questions?

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