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Foundation Mathematics

Topic 4 – Lecture 3: Introduction to Differential Calculus

Identifying Key Features of Gradients

Maximum and Minimum Points

Scope and Coverage

This topic will cover:

- Plotting maximum and minimum turning points using graphical means
- Identification and application of maximum and minimum points using differentiation.

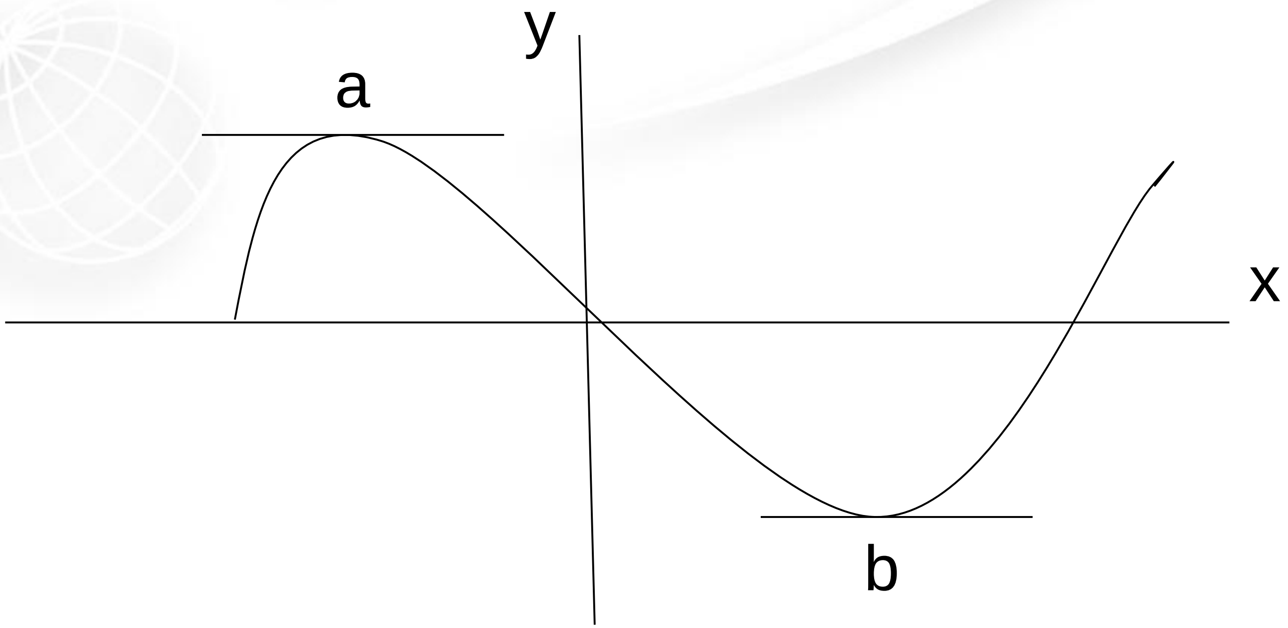
Learning Outcomes

By the end of this topic students will be able to:

- Plot maximum and minimum turning points using graphs
- Identify the maximum and minimum turning points using differentiation

Maximum and Minimum Points – Changing the shape of graphs - 1

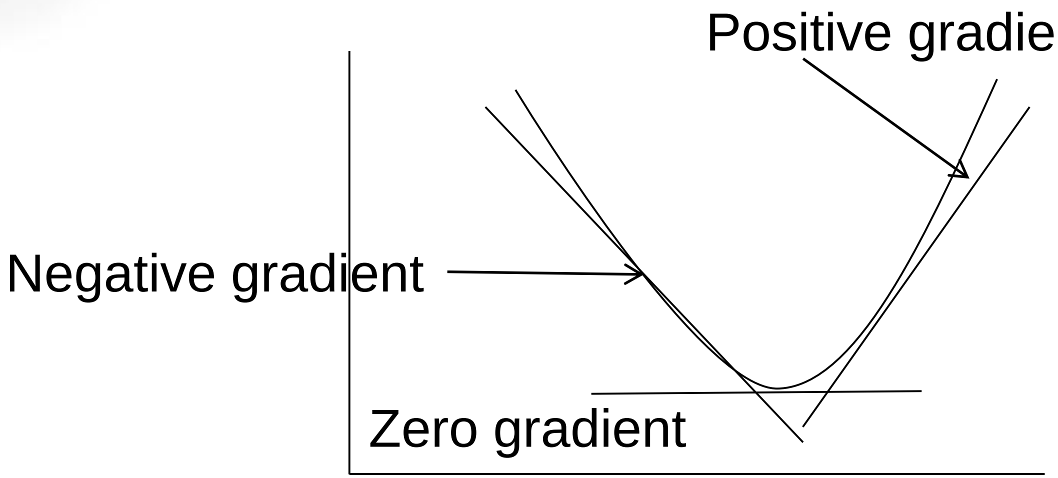
When we plot graphs of data it is often possible to identify changes in the shape of those graphs



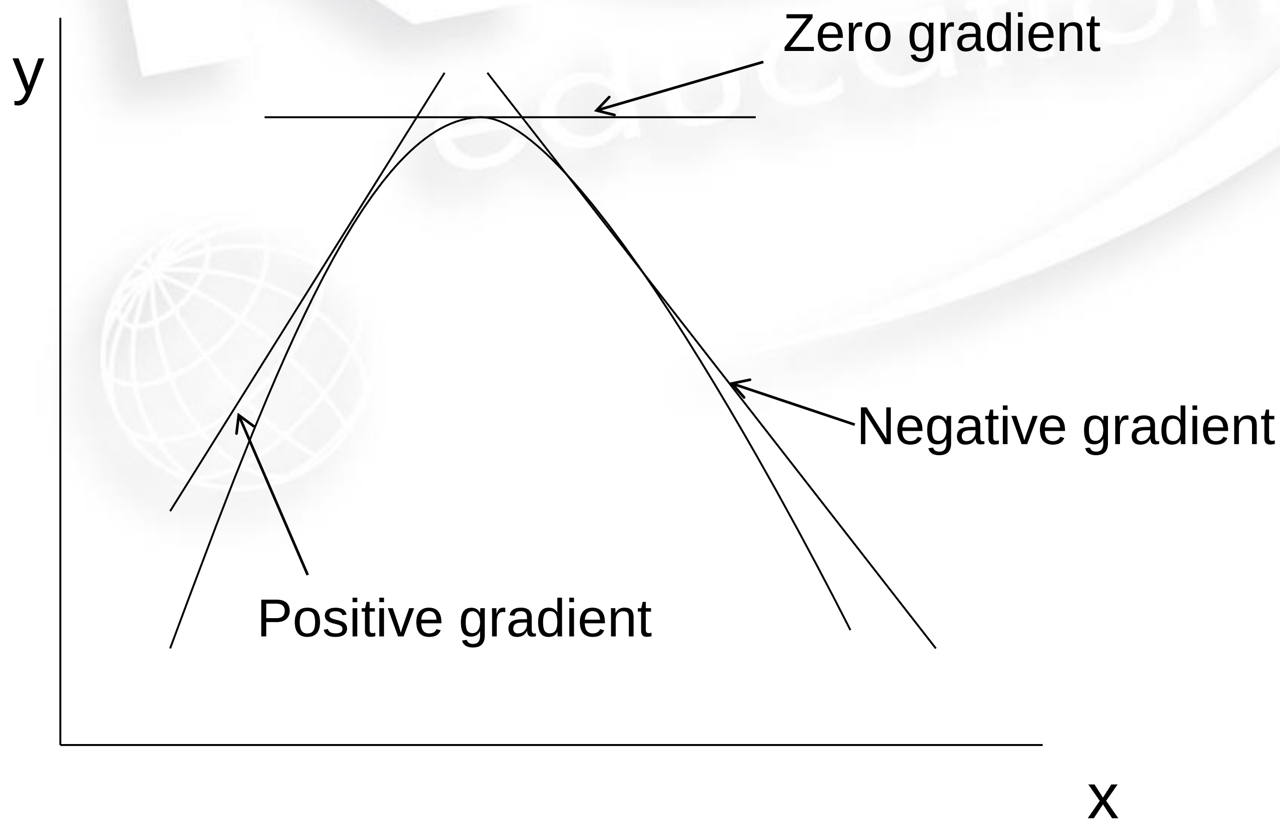
As we can see in this diagram the shape of the graph changes at point a and again at point b

Maximum and Minimum Points – Changing the shape of graphs - 2

- The shape of the graph may have changed for a number of reasons
- The change in shape of the graph is as a consequence of what we call **turning points**. These turning points are described as **maximum points** or **minimum points**

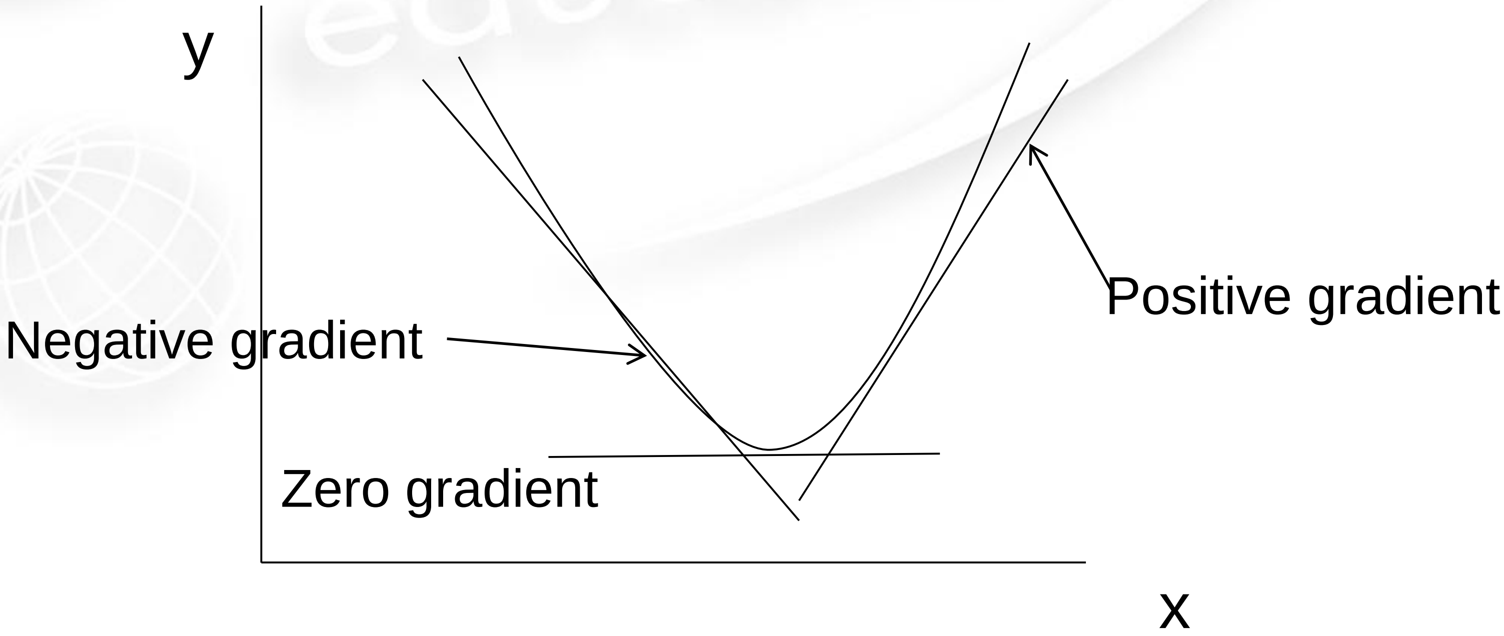


Maximum Turning Point



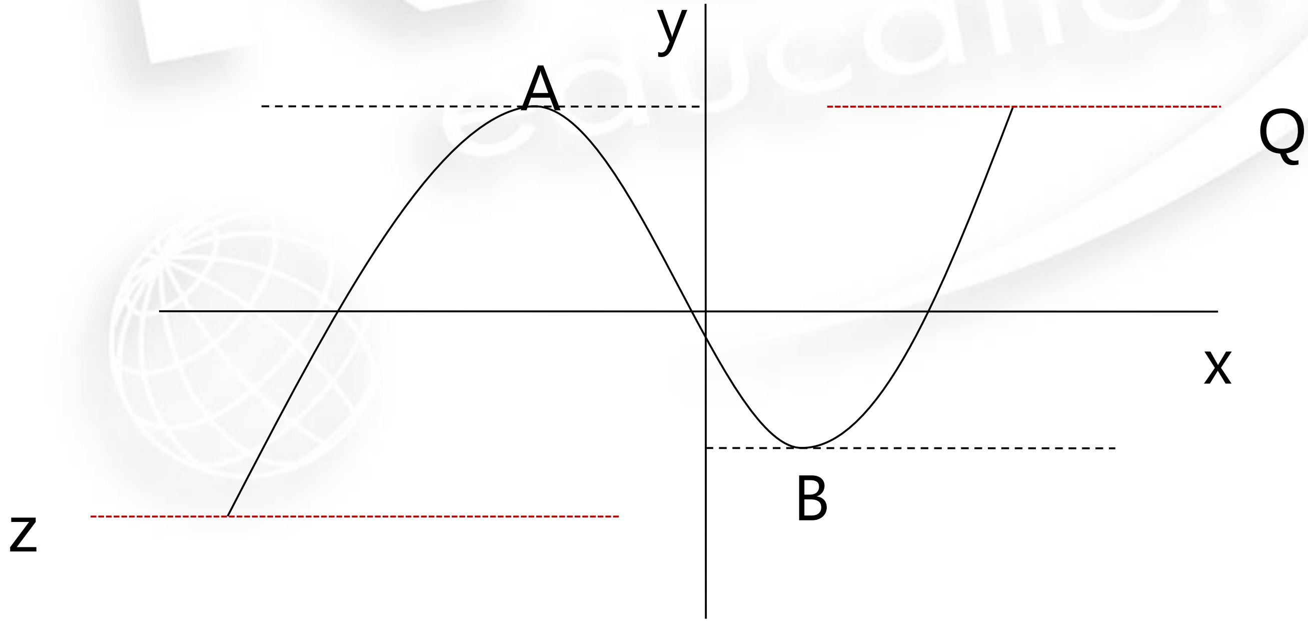
The shape of the graph shows the turning point at the highest value on the y axis, so this is a ***Maximum Turning Point***

Minimum Turning Point



The shape of the graph shows the turning point occurs at the lowest value on the y axis therefore is a ***Minimum Turning Point***

Complex Turning Points



- B is the **minimum turning point** as it has a **gradient of 0** and A is the **maximum turning point** as it also has a **gradient of 0**

Plotting Graphs - 1

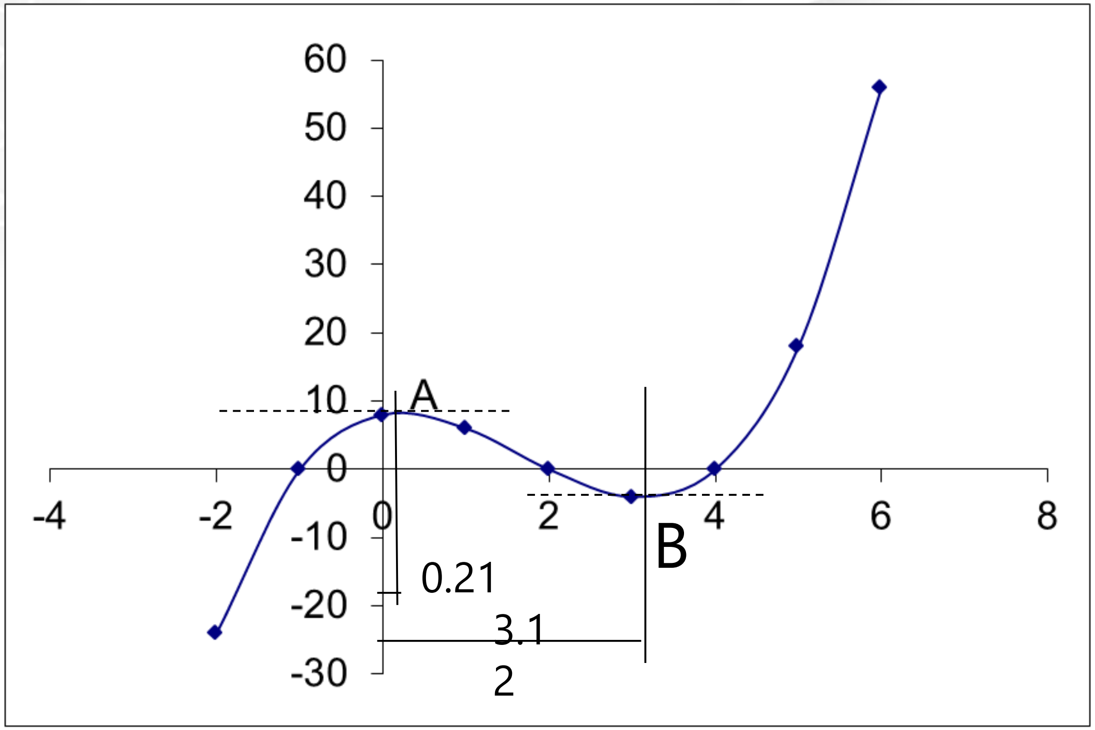
- Plot the graph of $y = x^3 - 5x^2 + 2x + 8$ for values of x between -2 and 6. Hence find the maximum and minimum values of y.
- To plot the graph we draw up a table

X	-2	-1	0	1	2	3	4	5	6
$y = x^3 - 5x^2 + 2x + 8$	-24	0	8	6	0	-4	0	18	56

- We can then plot this data as a graph to show the relationship between x and y

Plotting Graphs - 2

Plotting the graph gives us the following shape. It can be seen that neither is A the highest value nor B the lowest value but they are the turning points in respect of the values of y



Applying Differential Calculus - 1

- Consider the relationship between two variables y and x which can be expressed as $y = x^2 - 5$
- If we wish to identify any turning point in this relationship we can present it in a graph. To do this we need to plot values of x against y.
- Setting up a simple table requires us to propose limits for the values of x that we will use.

Applying Differential Calculus - 2

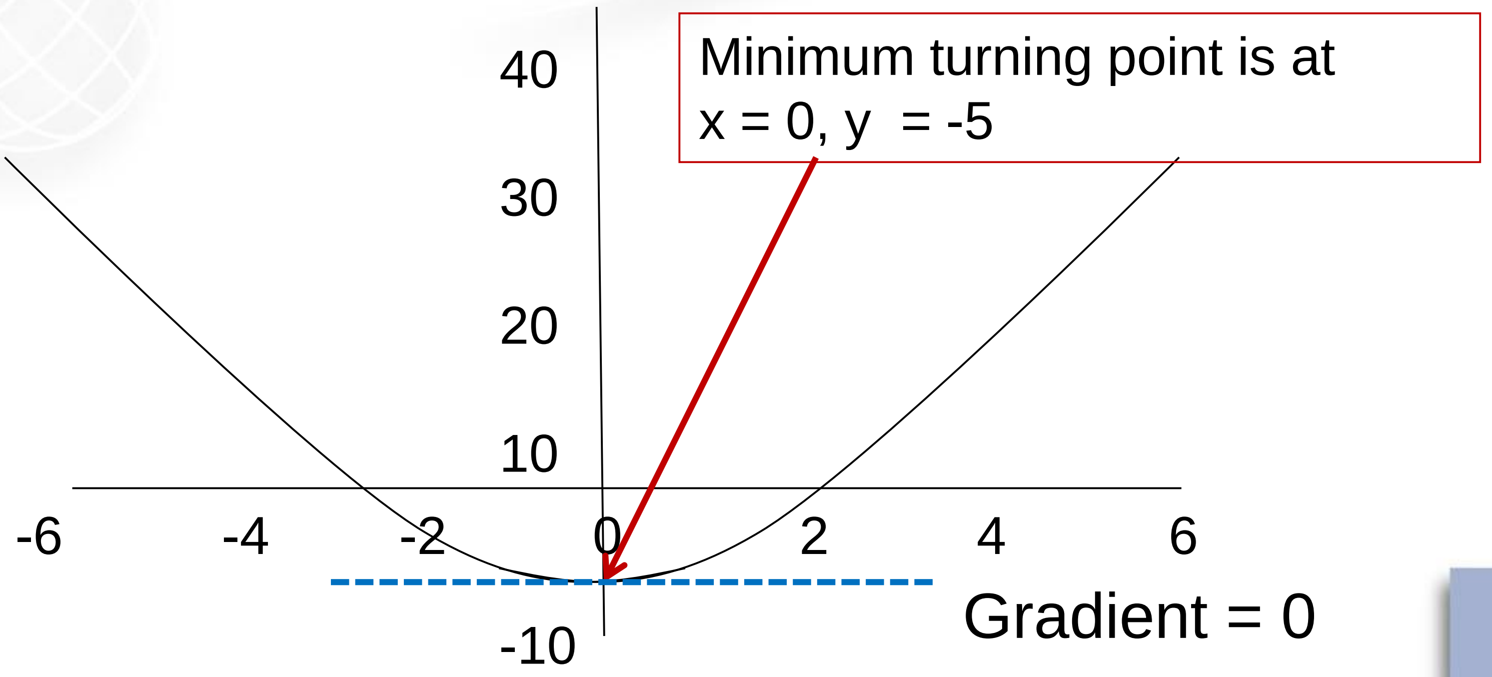
- This is presented as $-n \leq x \leq n$
- Therefore if we set limits of $-5 \leq x \leq 5$ we can now construct a table of values for both x and y

Drawing Gradient Functions

By substituting values of x into the equation for y we get the following

x	-6	-4	-2	0	2	4	6
$y = x^2 - 5$	31	11	-1	-5	-1	11	31

If we then plot this as a graph we get a shape like this



Alternative Approach

- We can also locate maximum and minimum turning points by differentiation.
- Consider the previous example in which $y = x^2 - 5$
- If we differentiate we get the gradient function of $\frac{dy}{dx} = 2x$
- When x = 0, $\frac{dy}{dx} = 0$ so 2x = 0. We have a turning point (x, y) at (0, - 5)
- This is what we found using the other, graphical, method.

Alternative Approach - 2

- We take a value which is smaller than the x value (0)

In this case we can take – 0.2

- As the gradient function is $\frac{dy}{dx} = 2x$ this gives us a value of -0.4 on the left hand side.

On the right hand side we take + 0.2 which gives us a value of = 0.4

The gradient is therefore **negative**, **reaches zero** then becomes **positive**. This is the character of a minimum turning point. For a maximum turning point the gradient is **positive**, **becomes zero** and then **negative**.

Further Application – Area Part 1

- A farmer has 100m of fencing to enclose a rectangular field. What is the maximum area that can be enclosed?
- In this problem we need to express the information in the form of an equation. Therefore let the length of the rectangle = x and the width = y
- We know that the perimeter (p) of a rectangle is found from $p = 2x + 2y$

Further Application – Area Part 1 Continued

- We also know that the area of a rectangle (a) is a = xy
- As we are maximising area (a) we need to have a as a function of x
- To do this we need to eliminate the other unknown (y) in order to calculate x

Further Application – Area Part 2

From the formula for the perimeter p = 2x + 2y we can rearrange to express y in terms of both p and x, thus

$y = \frac{1}{2}(p - 2x)$ which when we remove the brackets = $\frac{1}{2}p - x$

Therefore as a = xy this can now be written as $= x(\frac{1}{2}p - x) = \frac{1}{2}xp - x^2$

Differentiating $\frac{1}{2}xp - x^2$ gives us $\frac{da}{dx} = \frac{1}{2}p - 2x$. Remember at a turning point – in this case our maximum point – the gradient function $\frac{da}{dx}$ is = 0

Further Application – Area Part 2 Continued

Therefore at a maximum point $\frac{1}{2}p - 2x = 0$

Rearranging we get $x = \frac{1}{4}p$ therefore $x = \frac{1}{4}$ the length of the perimeter

x = 25 metres

Further Application – Area Part 3

Substituting this value of x (25m) into our equation $y = \frac{1}{2}p - x$ we now get

$y = \frac{1}{2}p - \frac{1}{4}p$ rearranging this equation by subtraction gives $y = \frac{1}{4}p$

Therefore the value of y is also $\frac{1}{4}p$ (quarter of the perimeter) and therefore 25 metres

This shows that the length of the rectangle and width of the rectangle are the same – that it is a square.

The total area enclosed is therefore 25 metres x 25 metres = 625m^2

Topic 4 - Introduction to Differential Calculus 3

Any Questions?

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