Foundation Mathematics

Topic 4 – Lecture 1: Introduction to Differential Calculus

Introduction to Differentiation

Scope and Coverage

This topic will cover:

- An introduction to calculus as way of explaining rates of change
- An introduction to the mathematical techniques used during differentiation

Learning Outcomes

By the end of this topic students will be able to:

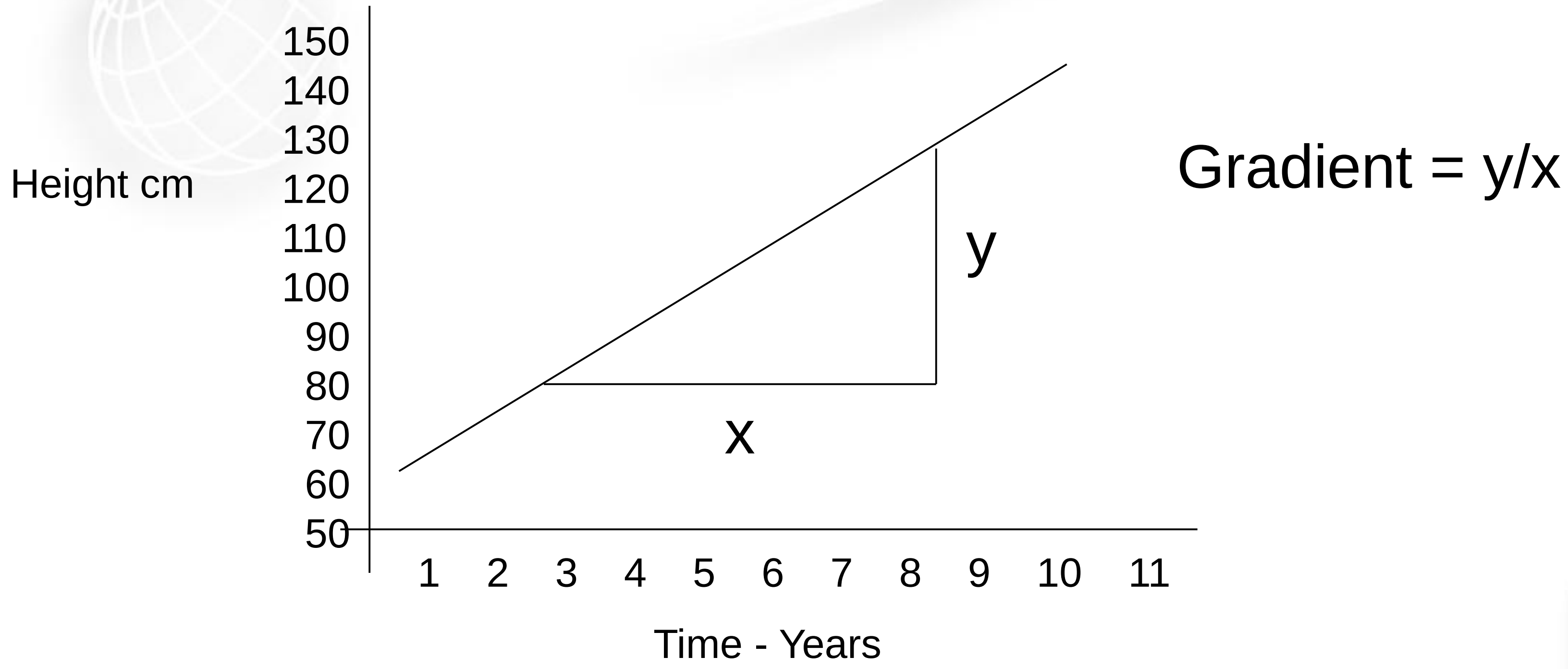
- Explain the rate of change of variables through differentiation
- Understand rates of change as expressed by a curve

Calculus and Change

- When we consider ***relationships*** between any two ***variables*** we are often required to consider:
  - What happens to the relationship if one of our ***variables*** should change.
  - An obvious example is that as we grow during our childhood years we get taller. Therefore our height ***changes*** over a period of time.
  - In this relationship our variables are ***time*** and ***height***

Calculus and Change – Straight Lines - 1

- If we consider this relationship between height and time it is possible to draw this information in a graphical form



Calculus and Change – Straight Lines 2

- This straight line relationship is expressed in the simple formula ***y = mx + c***
- This is very important as the ***gradient m*** is in fact the ***rate of change*** of ***y*** in relation to ***x***.
- In our example the gradient shows us how fast we are growing over time.

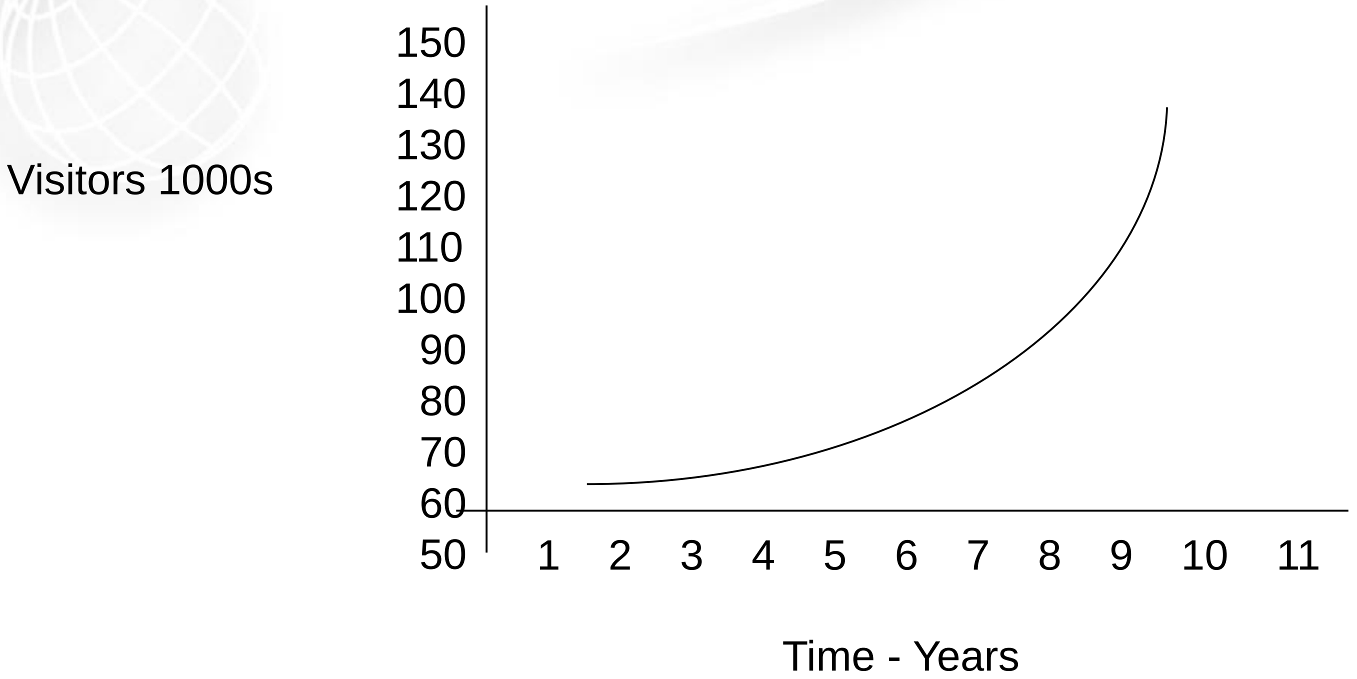
Calculus and Change - Curves

- Although the simple, straight line can show how we can determine rates of change, it is very unlikely in reality that relationships are easily identified as “straight lines”.

- More often we will gather data which when plotted on a graph will have a non linear or curved relationship between the variables.

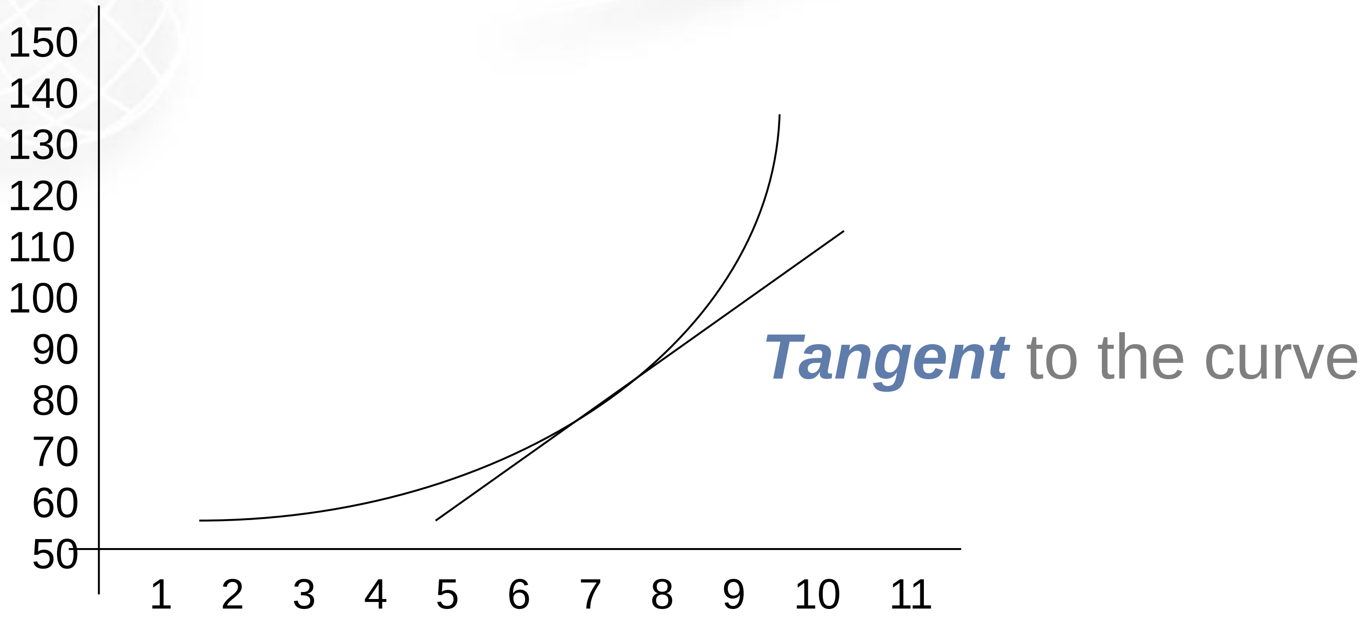
Plotting Curves

- The relationship between variables as a curve may be illustrated if we consider the interest in a new holiday destination



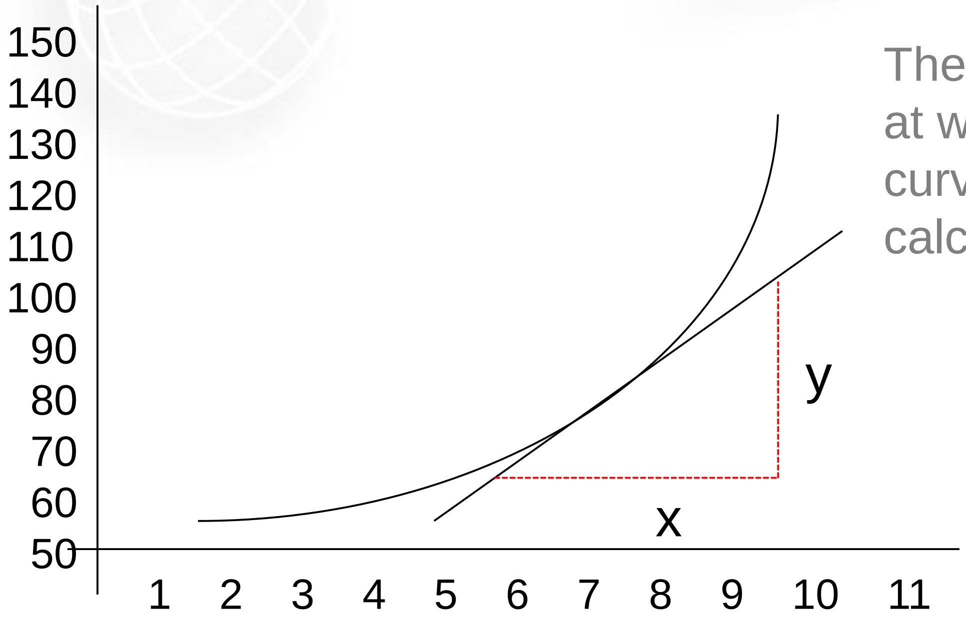
Tangent to the Curve

- This relationship shows clearly that the number of holiday visitors to the new destination does not follow a straight line. However the rate of change of this curve can be found by calculating the gradient of the curve.



Calculating the Gradient Using Tangents

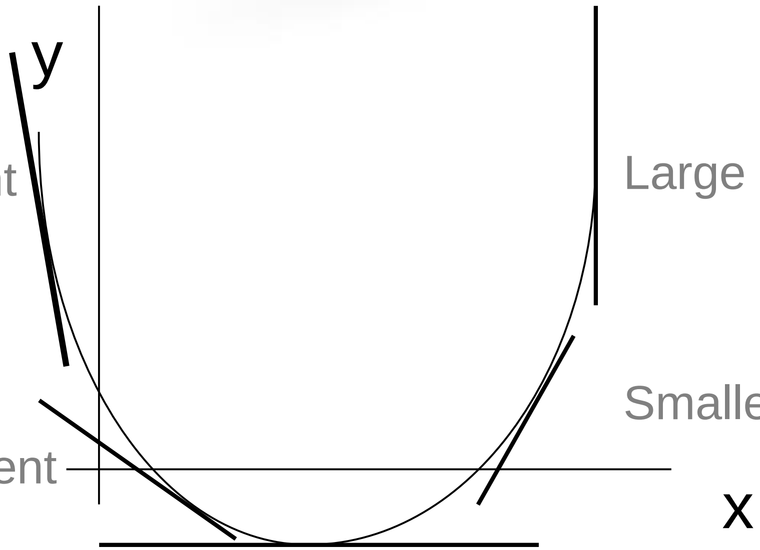
- In this example the ***gradient of the curve*** is calculated by drawing a ***tangent*** to the curve. A ***tangent*** is a straight line which ***touches*** the curve at a particular point. The ***gradient of the tangent*** is the same as the gradient of the curve at that ***particular point***.



The gradient of the curve at the point at which the tangent touches the curve can be found through calculating y/x

Gradients on a Curve

- Although this approach can be used it has a number of drawbacks
- It is often not accurate enough
- The gradient of curve will change depending upon where we place our tangent



Equation of a Curve

- Identifying rates of change through calculating the gradient of straight lines and the gradient of a curve it can be seen that the rate of change is as a consequence of the relationship between the two variables expressed as $\frac{y}{x}$
- However, as we have seen the gradient of a curve changes depending on where on the graph the tangent is drawn therefore $\frac{y}{x}$ is no longer suitable to express the gradient of a curve.
- When calculating the gradient of a curve we use the following equation to represent a curved relationship

$$y = x^n$$

Introduction to Differential Calculus

- The ***gradient*** of a curve at ***any*** point on the ***curve*** is given by its ***derived function*** thus if our equation for a curve is given as $y = x^n$ the derived function is $\frac{dy}{dx} = nx^{n-1}$
- This formula is true for all values of n including fractional and negative indices. The expression $\frac{dy}{dx}$, compares the rate of change of y with that of x
- dy is not a multiple of d and y and dy cannot be separated from dx
- The process of finding $\frac{dy}{dx}$ is called differentiation

Differentiation

- Examples of differentiation
- If we have a curve with a relationship $y = x^2$
- Differentiation of this curve using the formula $\frac{dy}{dx} = nx^{n-1}$ gives us an answer of $y = 3x^2$
- As we can see the initial power of x^3 is reduced by a factor of 1 to x^2 whilst the 3 now becomes the coefficient of x that is 3x

Differential Calculus Examples - 1

- If $y=\frac{1}{x}$ this can be also written as $y=x^{-1}$ and therefore if we differentiate using the formula we get $\frac{dy}{dx}=-x^{-2}$ which can be written as $=-\frac{1}{x^2}$
- If we consider $y=\sqrt{x}$ then this can be written as $y=x^{\frac{1}{2}}$
- By differentiation we get $\frac{dy}{dx}=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2x^{\frac{1}{2}}}=\frac{1}{2\sqrt{x}}$

Differential Calculus Examples - 2

- When a power of x is multiplied by a constant, the constant remains unchanged by the process of differentiation.

$$y = ax^n, \frac{dy}{dx} = anx^{n-1}$$

- Hence if

$$y = 3x^4, \frac{dy}{dx} = 3 \times 4x^3 = 12x^3$$

- This can be seen by example

Differential Calculus – Sum of Terms

- To differentiate an expression containing a sum of terms we differentiate each individual term separately.
- Example: $y = 3x^2 + 2x + 3, \dfrac{dy}{dx} = 3 \times 2x^1 + 2 \times 1x^0 + 0 = 6x + 2$
- As can be seen in this example x^0 as with all values raised to the power 0 is = 1
- The numerical value of + 3 at the end of the initial equation is eliminated through differentiation

Introduction to Differential Calculus

- When dealing with this form of equation it is possible to consider the following approach which would be found by differentiation.

- If $y = ax^3 + bx^2 + cx + d$ in which a, b, c and d are constants

- Differentiation would give us an expression thus:

$$\frac{dy}{dx} = 3ax^2 + 2bx + c$$

Topic 4 – Introduction to Differential Calculus 1

Any Questions?

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