Bringing British Education to You

www.nccedu.com

Foundation Mathematics

Topic 3 - Lecture 2: Solving algebraic equations using graphs

Presenting quadratic equations in graphical form

Solving simultaneous equations using graphs

V1.0

Scope and Coverage

This topic will cover:

- The presentation of quadratic equations as graphs
- Solving simultaneous equations using graphs

Learning Outcomes

By the end of this topic students will be able to:

- Present a range of quadratic equations in graphical form
- Solve simultaneous equations using graphs

Drawing Graphs of Quadratic Equations - 1

- $ax^2 + bx + c$ where a, b and c are constants is a quadratic function of x

- When plotted, quadratic functions always give a smooth curve known as a ***parabola*** (or a ***parabolic curve***)

Drawing Graphs of Quadratic Equations - 2

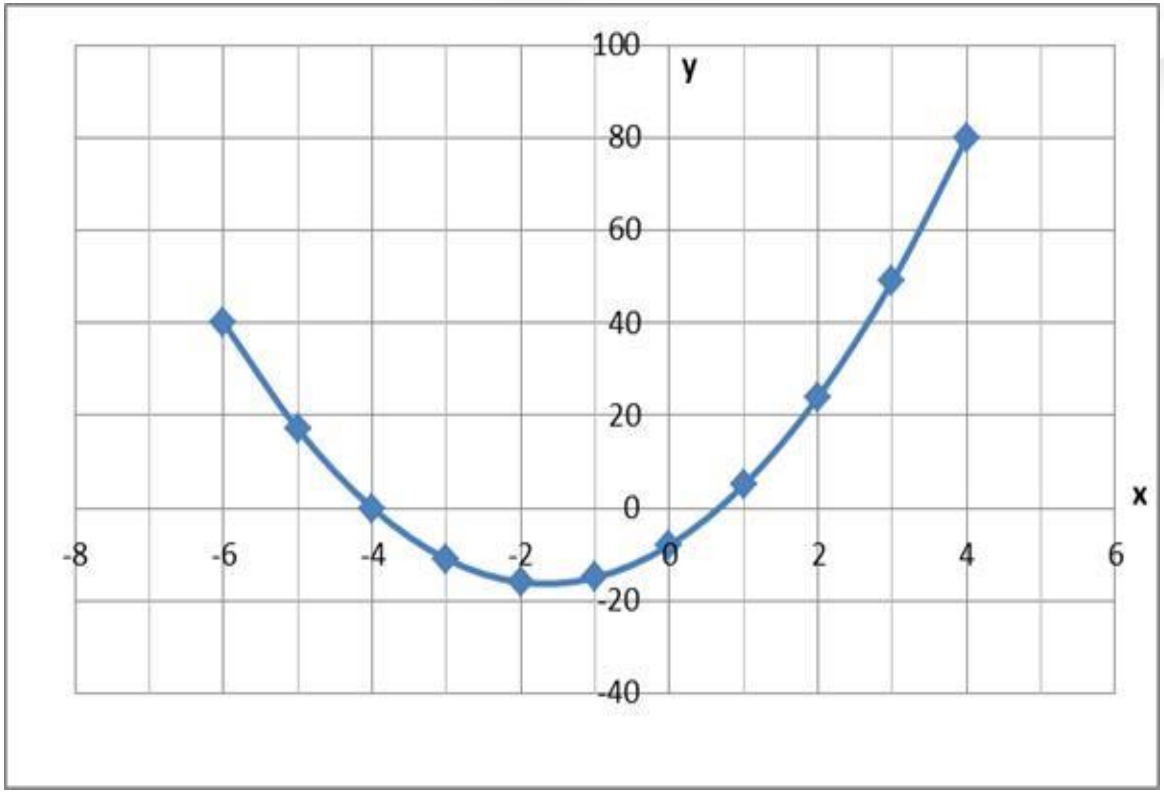
- ***Example***
- Plot the graph of $y = 3x^2 + 10x - 8$ between x=-6 and x=4
- As we have done before it is easiest to set out a table of values for x and then calculate the corresponding values for y
- Therefore for our range of x values we get:

x	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**
$3x^2$	108	75	48	27	12	3	0	3	12	27	48
10x	-60	-50	-40	-30	-20	-10	0	10	20	30	40
-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8
y	40	17	0	-11	-16	-15	-8	5	24	49	80

Remember we only plot values of x against y

Drawing Graphs of Quadratic Equations - 3

By plotting our values of x against those calculated for y we get the following graph:



Quadratic graphs always give this curved shape.

Drawing Graphs of Quadratic Equations - 4

- By understanding how we can plot information on a graph it is possible to solve quadratic equations using this method. This can provide a useful way of solving equations and presenting information in accessible ways.
- Example:

  Plot the graph of $y = 6x^2 - 7x - 5$ between x=-2 and x=3. And therefore solve the equation $6x^2 - 7x - 5 = 0$
- We can of course solve the equation by factorising but graphs can be more fun

Drawing Graphs of Quadratic Equations - 5

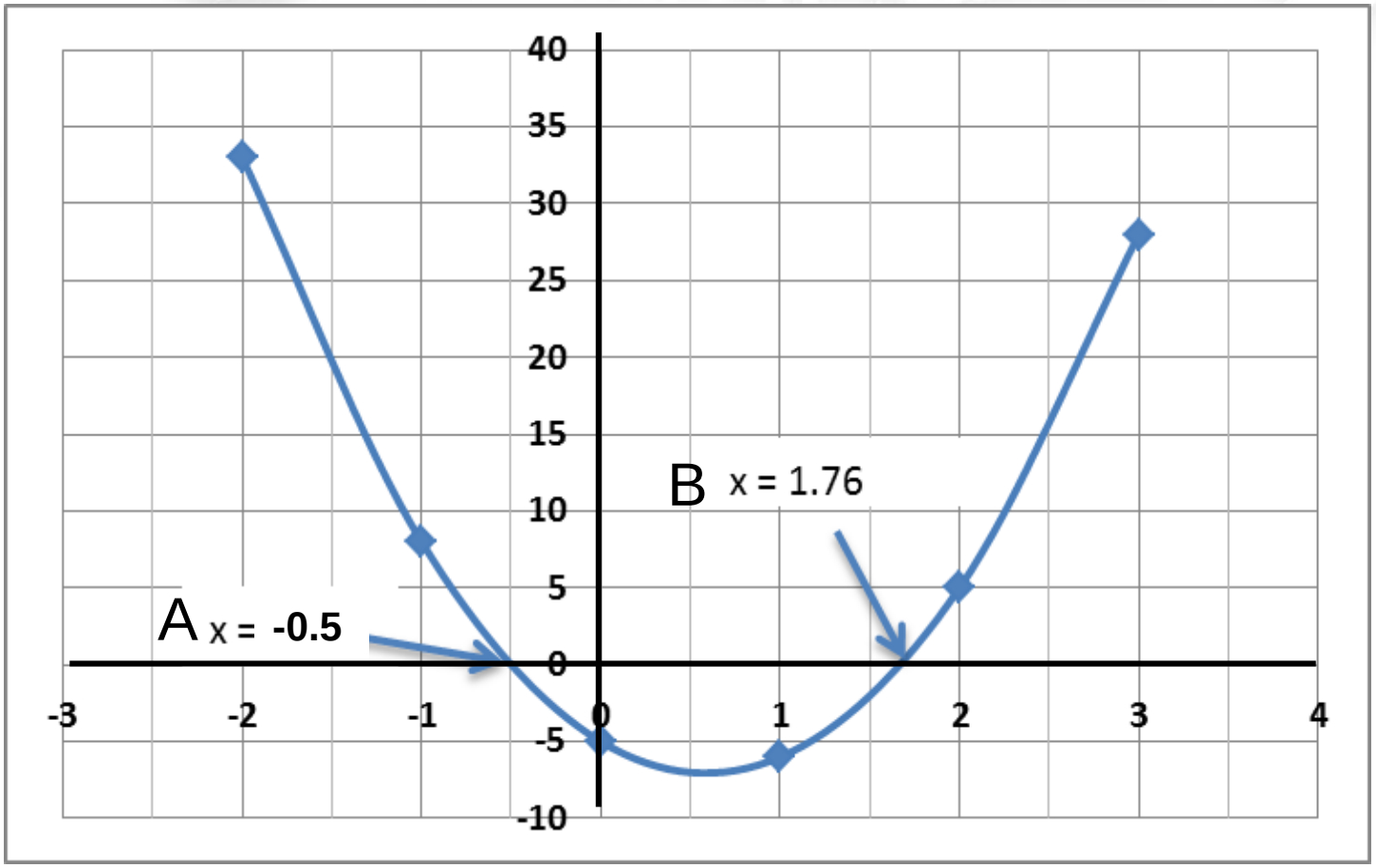
- Example continued:
- The first thing we need to do is to set up our table of values

x	-2	-1	0	1	2	3
y	33	8	-5	-6	5	28

- To solve the equation $6x^2 - 7x - 5 = 0$ we have to find the values of x when y=0. That is, we have to find the values of x where the graph cuts the x-axis

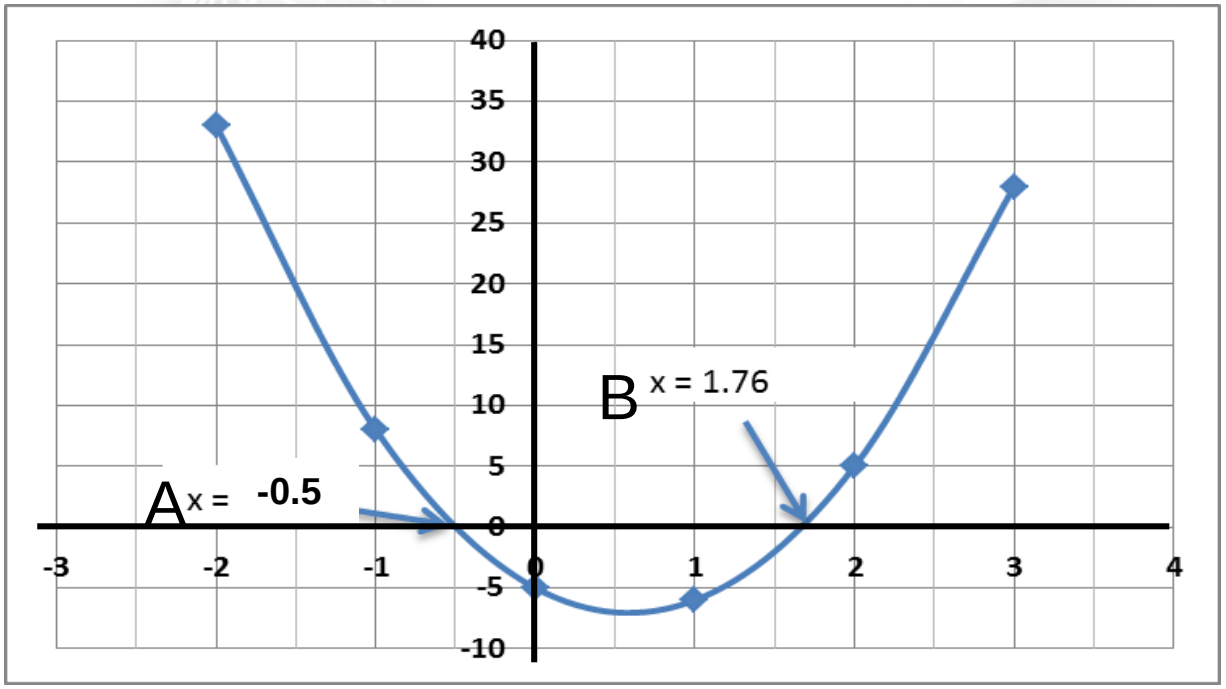
Drawing Graphs of Quadratic Equations - 6

- The graph of our equation appears as:



Drawing Graphs of Quadratic Equations - 7

- Once we have our graph it is easy to see the points on the graph when y = 0. These are shown by points A and B. We can then read the corresponding numerical value on the x axis.



Therefore the values of x are

x = -0.5 or

x = 1.76

Graphical Solution for Simultaneous Equations - 1

- Although it is possible to solve simultaneous equations using techniques discussed in our previous unit, it is also possible for this to be achieved using graphs.
- This approach will be illustrated by example:
- Solve graphically $y-2x=2$ …….. 1

  $3y+x=20$ ……..2
- To simplify these expressions
- Equation 1 can be written as $y = 2 + 2x$
- Equation 2 can be written as $y = \frac{20 - x}{3}$

Graphical Solution for Simultaneous Equations - 2

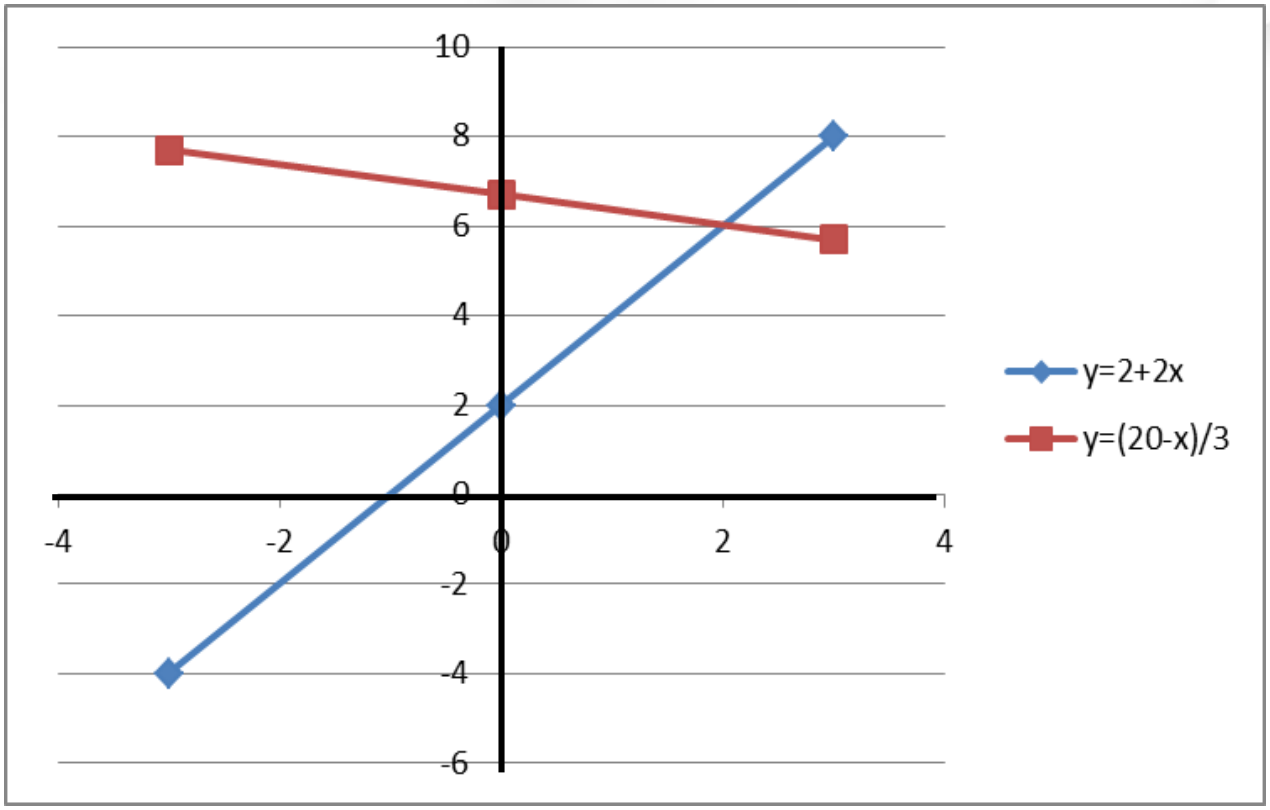
- By placing information into a table (as we have done for single equations) we can plot the two equations on the same axes.

x	-3	0	3
$y = 2 + 2x$	-4	2	8
$y = \frac{20 - x}{3}$	7.7	6.7	5.7

- Now that we have this information we can plot both values of y against x on the same graph

Graphical Solution for Simultaneous Equations - 3

- The solutions of the equations are the co-ordinates of the point where the two lines cross



As we can see the graphs **intersect** at the point where x = 2 and where y = 6

Graphical Solution for Simultaneous Equations - 4

- The solutions for the equations

$$y - 2x = 2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$
$$3y + x = 20 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

- Are $x = 2$ and $y = 6$
- To check this we can substitute values into our equations
- From equation 1 we get
  - $6 - 2 \times 2 = 2$ ……..which is correct
- From equation 2 we get
  - $18 + 2 = 20$ ……..which is correct

Graphical Solution for Simultaneous Equations - 5

- A further example:
- Draw the graph of $y = (3+2x)(3-x)$ for values of x from $-1\frac{1}{2}$ to 3
- On the same axes, and with the same scales, draw the graph of $3y = 2x + 14$
- From your graphs determine the values of x for which $3(3+2x)(3-x) = 2x + 14$

Graphical Solution for Simultaneous Equations - 6

- To plot the graph of $y = (3+2x)(3-x)$ we draw up the following table

x	-1½	-1	-½	0	½	1	1½	2	2½	3
$y = (3+2x)(3-x)$	0	4	7	9	10	10	9	7	4	0

- The equation $3y = 2x + 14$ may be written as $y = \frac{2x+14}{3}$
- To draw this graph we need only take three points since it is a linear equation.

Graphical Solution for Simultaneous Equations - 7

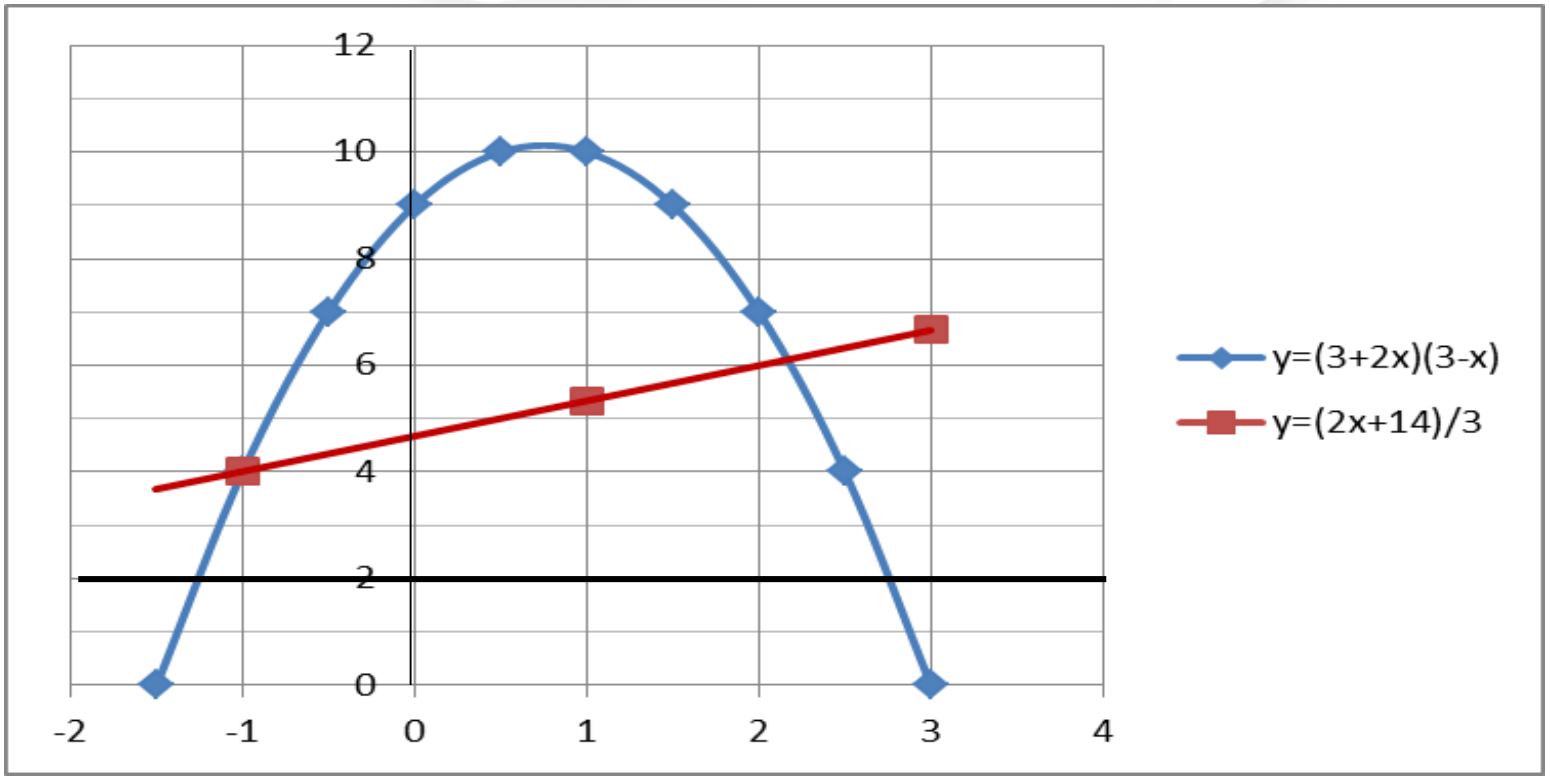
Plotting values for the equation $y = \frac{2x+14}{3}$ requires us to prepare a table of values

X	-1	1	3
$y = \frac{2x + 14}{3}$	4	5⅓	6⅔

Now that we have the values for both our equations we may plot these graphically

Graphical Solution for Simultaneous Equations - 8

Plotting the graph of our equations gives us the following shape:



Graphical Solution for Simultaneous Equations - 9

- Since we were asked to find the co-ordinates of x when the equation $3(3+2x)(3-x)=2x+14$
- Remember this equation may be written as

$$(3+2x)(3-x)=\frac{2x+14}{3}$$

- The co-ordinates where the curve and the line intersect therefore give us the values when

$$(3+2x)(3-x)=\frac{2x+14}{3}$$

- And thus give the solutions which are $x=-1$ and $x=2.17$

Topic 3 – Solving Algebraic Equations using Graphs 2

Any Questions?

Bringing British Education to You

www.nccedu.com