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Foundation Mathematics

Topic 2 – Lecture 2: Using Algebraic Equations

Solving Simple Quadratic Equations

Solving Simultaneous Equations

V1.0

Scope and Coverage

This topic will cover:

- An introduction to solving simple quadratic equations
- An introduction to solving simultaneous equations

Learning Outcomes

By the end of this topic students will be able to:

- Recognise quadratic expressions and solve quadratic equations using a range of techniques
- Solve a range of simultaneous equations through a range of techniques

Quadratic Equations - 1

- Typical examples of quadratic equations often take the form $ax^2+bx+c=0$
- x^2 signifies that this is a quadratic equation
- $x^2+8 = 0$ (square)
- $x^2 + 4x -12 =0$ (square and first power)

Quadratic Equations - 2

- $ax^2+bx+c=0$
  - It is possible for $b=0$, therefore our equation becomes simply $ax^2+c=0$
- Consider $x^2 - 81 = 0$,
  - Simply transposing the equation gives $x = 81$
  - To calculate a value for x we need only to take the square root of both sides, therefore $\sqrt{x^2} = \sqrt{81}$
  - Any positive number has both a positive and a negative root.
  - Therefore, $x = \pm 9$

Solution by Factors - 1

- If the product of two factors is zero, then one factor or the other factor must be zero or they may both be zero.
- Thus if $ab = 0$, then either $a = 0$ or $b = 0$ or both $a = 0$ and $b = 0$. We make use of this fact in solving quadratic equations.
- Solve the equation $x^2-5x+6 = 0$
- Factorising, $(x-3)(x-2) = 0$
- Therefore either $x – 3 = 0$ giving $x = 3$ or $x – 2 = 0$ giving $x = 2$

Solution by Factors - 2

- Solve the equation $6x^2+x - 15 = 0$
  - Factorising $(2x-3)(3x+5)=0$
  - either $2x-3=0$ giving $x = 3/2$
  - or $3x+5 = 0$ giving $x = -(5/3)$
  - The solutions are $x = 3/2$ or $x = -(5/3)$

Solution by Formula - 1

- The standard form of the quadratic equation is $ax^2+bx+c=0$
- It can be shown that the solution of this equation is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Note that the whole of the numerator including –b is divided by 2a. The formula is used when factorisation is not possible.

Solution by Formula - 2

- Solve the equation: $3x^2 + 8x + 2 = 0$
  - Comparing with our generic expression of a quadratic equation $ax^2+bx+c=0$
  - we have $a = 3$, $b = 8$ and $c = 2$. Substituting into the formula for solving the quadratic equation we have:

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 3 \times 2}}{2 \times 3}$$

$$\frac{-8 \pm \sqrt{64 - 24}}{6}$$

Solution by Formula - 3

$= \dfrac{-8 \pm \sqrt{40}}{6} = \dfrac{-8 \pm 6.325...}{6}$ either x $= \dfrac{-8 + 6.325...}{6}$ or $\dfrac{-8 - 6.325...}{6}$

Therefore x = -2.39 or -0.28 (rounded to 2 decimal places)

Solving Quadratic Equations - 1

- Solve the equation $\frac{3}{2x-3}-\frac{2}{x+1}=5$

- To solve this we must first treat it as an algebraic fraction and multiply throughout by the lowest common multiple. This then gives:

$$3(x+1)-2(2x-3)=5(2x-3)(x+1)$$

Solving Quadratic Equations - 2

- Which through simplification gives us:

$$3x + 3 - 4x - 6 = 5(2x^2 - x - 3)$$

$$-x + 9 = 10x^2 - 5x - 15$$

$$10x^2 - 4x - 24 = 0$$

- Which ultimately gives us the form of a quadratic equation which can be solved by applying the formula.

Solving Quadratic Equations - 3

- From our process of simplification we can see that

$$10x^2 - 4x - 24 = 0$$

a = 10, b = -4 and c = -24

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 10 \times (-24)}}{2 \times 10}$$

- Therefore…

Quadratic Equations - 12

$$\frac{4 \pm \sqrt{16+960}}{20} = \frac{4 \pm \sqrt{976}}{20} = \frac{4 \pm 31.24}{20}$$

$$x = \frac{4+31.24...}{20} \quad \text{or} \quad x = \frac{4-31.24...}{20}$$

- Therefore $x = 1.762$ or $x = -1.362$

  Answers given to 3 decimal places

Simultaneous Equations - 1

- Consider these two equations:

$$2x + 3y = 13$$
$$3x + 2y = 12$$

- Each equation contains two unknown quantities x and y.

Dealing with both these equations together it is possible to identify the numerical values of both x and y.

Simultaneous Equations - 2

- Solve the equations $3x + 4y = 11$

  $x + 7y = 15$

- In order to proceed we need to number the equations (1) and (2)
- Thus… $3x + 4y = 11$ ……(1)

  $x + 7y = 15$ ………(2)

- If we multiply equation (2) by a 3 we get the same numerical value for x in both equations hence

  $3x + 4y = 11$ …………(1)

  $3x + 21y = 45$…………..(3)

Simultaneous Equations - 3

- Subtracting equation (3) from equation (1) gives $17y = 34$, it follows therefore that $y = 2$

- To find a value of x we substitute back into eq. (1)
  $3x + 8 = 11$

  $x = 1$

- Solution: $x = 1, y = 2$

Simultaneous Equations - 4

- Solve the equations $\frac{2x}{3} - \frac{y}{4} = \frac{7}{12}$ ....(1)

  $\frac{3x}{4} - \frac{2y}{5} = \frac{3}{10}$ ....(2)

- The first thing to do here is simplify and the best way is to remove the fractions by multiplying both equations by their respective lowest common multiple which in the case of Eq. (1) is 12 and for Eq. (2) is 20

Simultaneous Equations - 5

- This gives: $8x – 3y = 7$……….3)
  $15x – 8y = 6$………4)
- To get identical values of either x or y we look to the coefficient of either of these terms to determine which are most straightforward to be eliminated. In this case it would appear that multiplying Eq. 3 by 8 and Eq. 4 by 3 will give an identical value of y

Simultaneous Equations - 6

- Multiplying equation (3) by 8, $64x – 24y = 56$……….(5)
- Multiplying equation (4) by 3, $45x – 24y = 18$……….(6)
- We can now subtract Eq. 6 from Eq. 5
- This gives us $19x = 38$ therefore $x = 38/19 = 2$
- Substituting into equation 3) which originally looked like this $8x – 3y = 7$
- We get $16 - 3y = 7$ therefore $3y = 9$ or $y = 3$
- Solution: $x = 2, y = 3$

Simultaneous Equations - 7

- Applying to real life situations can help to illustrate the application of simultaneous equations in solving for multiple unknowns
- Example
  - A manager and 7 labourers together earn $650 per week while two managers and 17 labourers together earn $1525 per week. Find the weekly wages of a manager.
  - Let a manager earn $x per week and a labourer earn $y per week.
  - Writing these as equations we can show the relationship between the unknowns and the numerical values

$$x + 7y = 650 \quad \ldots\ldots\ldots\ldots(1)$$

$$2x + 17y = 1525 \quad \ldots\ldots\ldots\ldots(2)$$

Simultaneous Equations - 8

- To eliminate one of the unknowns we look to establish how we can most easily find a common value for one of the unknowns. In this case we can do this by multiplying equation (1) by 2 this then gives us

$$2x + 14y = 1300 \ldots\ldots\ldots\ldots(3)$$

- If we then subtract equation (3) from equation (2) $2x + 17y = 1525$
- $3y = 225$, which then gives us $y = 75$

Simultaneous Equations – 9

- By substituting y = 75 in equation (1), we get

$x + 7 \times 75 = 650$

$x + 525 = 650$

$x = 125$

- Therefore a manager earns $125 per week.

Simultaneous Equations - 10

- Consider the following
  - $900a + 30b + c = 500$…………..(1)
  - $1600a + 40b + c = 800$…………(2)
  - $2500a + 50b + c = 900$…………(3)
- Again we have a combination of values for both unknowns plus numerical values attached to each equation.
- Same approach - elimination of unknowns and substitution.

Simultaneous Equations - 11

- The first approach here is to eliminate the unknown with the lowest coefficient which in this case is c

- To do this we subtract equation 1 from equation 2 this then gives us

$$700a + 10b = 300 \ldots\ldots\ldots(4)$$

Simultaneous Equations - 12

- We need to further eliminate c from the other equations and for this we subtract equation 2 from equation 3 this gives us

$$900a+10b=100 \quad \ldots\ldots\ldots\ldots(5)$$

- Now we have to eliminate one of the unknowns from both Eq. 4 and 5
- To do this we subtract Eq. 5 from Eq. 4 which gives us

$$-200a = 200 \text{ therefore } a = -1$$

Simultaneous Equations - 13

- Once we have achieved this we need to substitute -1 for a in one of our equations in this case we will take equation (5) $900a + 10b = 100$

- Substitution gives us $-900 + 10b = 100$

$$10b = 1000$$

$$b = 100$$

Simultaneous Equations - 14

- Finally substituting back into Eq. 1

  $900a + 30b + c = 500$ now looks like…

  $-900 + 3000 + c = 500$, by transposition we get

  $c = 500+900-3000 = -1600$

- Therefore we have solved three simultaneous equations and identified three unknown quantities.

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Any Questions?

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